Black holes and quantum information

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The increasing entropy of a black hole that evaporates by emitting Hawking radiation is at odds with the predictions of quantum mechanics. Juan Maldacena discusses the latest advances in solving this puzzle, known as the black hole information paradox.

Black holes are fascinating objects where the geometry of spacetime is deformed in such a drastic way that an entire spacetime region is invisible to an outside observer. This invisible region is called the 'interior' of the black hole. The visible one is the 'exterior'. The two are separated by the black hole 'horizon'. Over the past few years, black holes have made the headlines thanks to exciting observations from the LIGO/Virgo gravitational wave observatories and the Event Horizon Telescope. There has also been remarkable progress on understanding some theoretical aspects of black holes centred on their connection with quantum mechanics.

Let's first recall some older results on the quantum behaviour of black holes such as Stephen Hawking's prediction that black holes should emit thermal radiation. Although this radiation is too small to be detected from astrophysical black holes, it poses very interesting theoretical puzzles, which stem from a conflict between the expectations from quantum theory versus those from gravity.

The most famous of these conundrums is the information paradox, which is based on the following thought experiment. Imagine a black hole formed by concentrating a sufficient amount of matter. The black hole will emit Hawking radiation and gradually lose energy. Eventually, it will become very small and probably disappear altogether (evaporate). Through this process, the energy contained in the matter that collapsed into the black hole comes out again carried by the radiation. In itself, this is not too surprising; when a star forms some of the energy also comes out as radiation, or if the star eventually explodes as a supernova, in the form of the various chemical elements that we have in nature. But black holes are special, because this radiation comes from the empty space region near the black hole horizon and it seems to be independent of the matter that had formed the black hole. In addition, Hawking radiation seems to be thermal, so that the entropy of the region outside the black hole is increasing.

You might think: of course the entropy increases, this is just the second law of thermodynamics. However, the entropy we are used to in thermodynamics is a 'coarse-grained' entropy, meaning that it arises because we cannot keep track of all the variables of the problem. But, in a theory that preserves information, such as quantum mechanics, there is a second notion of entropy, the 'fine-grained entropy', or von Neumann entropy. This entropy characterizes the fundamental ignorance that we have about the state of the system, even if we can do arbitrarily complicated measurements. This entropy cannot increase for a system subject to ordinary (unitary) evolution in a quantum theory. But, according to Hawking, this entropy does increase when a black hole forms and evaporates because the process involves an entangled pair of particles coming out of the vacuum, with one member of the pair going into the black hole interior and the other going out as Hawking radiation. As the particles going into the black hole interior are fundamentally inaccessible, the entropy of the outside grows.

In order to solve this puzzle, it is important to understand how to compute entropy in a theory of gravity. The first classic result is that the thermodynamic entropy of a black hole (S_{BH}) is proportional to the area of its horizon ($(\mathrm{Area})_{\mathrm{hor}}$),

$$S_{\mathrm{BH}} \sim \frac{(\mathrm{Area})_{\mathrm{hor}}}{4 l_p^2} \qquad (1)$$

where l_{p} is the Planck length.

This formula follows from the first law of thermodynamics $\mathrm{d}E = T\,\mathrm{d}S$, where E is the energy, S the entropy and T the temperature, and Hawking's formula for the temperature. More precisely, the full entropy is the area plus the von Neumann entropy of the matter outside the black hole (S_{matter}), including the entropy of the quantum fields outside the black hole.

$$S_{\mathrm{BH}} = \frac{(\mathrm{Area})_{\mathrm{hor}}}{4 l_p^2} + S_{\mathrm{matter}} \qquad (2)$$

This formula is the coarse-grained entropy of the black hole and it obeys the second law of thermodynamics[1].

During the past 15 years a new gravitational entropy formula has been developed. It computes the fine-grained entropy of a black hole. This formula started with work by Shinsei Ryu and Tadashi Takayanagi[2], and was further improved by many others. It also involves an area, but that of a different surface! In order to compute

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the fine-grained entropy of a black hole, imagine that it is surrounded by a large surface. Then one needs to compute the area of this surface and add the entropy of quantum fields outside, in a combination similar to the one in Eq. 2. Now, shrink the surface until this sum has a minimal value. This is the fine-grained (von Neuman) entropy (S_{vN}). In other words

$$S_{\mathrm{vN}} = \min \left[\frac{(\mathrm{Area})}{4l_p^2} + S_{\mathrm{matter}} \right] \tag{3}$$

Importantly, one is allowed to take this surface to the black hole interior when searching for the minimum.

For the case of a black hole resulting from the collapse of a star, the minimum occurs when the surface is shrunk all the way to the centre of the star, where the area term is zero. The centre of the star lies in the interior of the black hole and in this case the entropy arises solely from the matter term in Eq. 3, from entropy of the star, in agreement with unitarity of quantum mechanics, which requires the fine-grained entropy to be constant.

As the black hole evaporates, both its fine-grained entropy and the entropy of the Hawking radiation increase up to a maximum of the order of the area of the initial black hole, owing to the creation of entangled particle pairs. However, if the system was described by a unitary quantum mechanics, the entropy can initially increase, but should eventually decrease back to zero.

Last year, a couple of very interesting papers[3,4] showed that this is indeed the case, provided the entropy is computed using the proper formula for the fine-grained entropy in Eq. 3. The crucial point is that for a black hole that has been evaporating for a long time, the entropy due to the entangled particle pairs becomes so high that the surface minimizing the entropy sits close to the horizon, as opposed to a surface that shrinks all the way to zero at the centre of the collapsing star. Owing to the effects of the evaporation of the black hole, this surface is shrinking, so that the von Neuman entropy of the black hole is shrinking in agreement with the expectations for a unitary quantum system. Furthermore, the proper way to compute the entropy of the radiation should also incorporate most of the interior region of the black hole. This region now includes both members of the entangled particle pair created by the Hawking mechanism, the one outside and the one inside, and therefore they do not contribute to the total entropy. In this way the entropy for the radiation is shrinking too.

This answers the question of what was wrong with Hawking's calculated entropy of radiation. He was not using the right formula for the fine-grained entropy for a gravitational system (which was not known at the time). We now have a definite method for computing the entropy in a way that agrees with the expectations from quantum mechanics. Furthermore, this computation is done according to gravitational rules. These papers, together with some further additional developments, resolved one aspect of the information paradox: they provided a formula for the entropy of the Hawking radiation that agrees with the expectations of a quantum theory.

This is the latest of a long series of developments that connect spacetime geometry to quantum information theory. von Neumann entropy is crucial to quantum information, in the same way that Shannon entropy is crucial to classical information. As we have seen, von Neumann entropy is related to areas, to a geometrical aspect of spacetime.

This exploration of the connections between spacetime geometry and quantum information theory also involves concepts from condensed matter, chaos, quantum computation and so on. A constant theme is that various phenomena in these fields are 'geometrized', meaning that they can be described in terms of features of the spacetime geometry. In particular, quantum entanglement is paramount for the emergence of the spacetime geometry.

Research in this area has been guided by concrete mathematical models for quantum systems that capture features of black holes. These quantum systems must have a series of necessary properties. For example, they should contain a number of qubits of the order of the black hole entropy. They should be strongly interacting and very chaotic. Several concrete quantum mechanical models displaying properties of black holes have been proposed, for example, quantum circuits that could be realized with trapped atoms or ions[5]. Simpler quantum systems display only some of the properties and more complex ones are believed to describe black holes in very concrete theories of Einstein gravity.

Experimental efforts are now underway to construct quantum systems that display the simplest properties. Questions that we hope to answer through these constructions are, for example, how generic is the emergence of gravity or what happens to spacetime when quantum corrections are fairly important?

There are now 'quantum gravity in the lab' conferences where theorists and experimentalists discuss possible ways to create quantum systems with properties analogous to those of black holes. A long workshop is now taking place at the Kavli Institute for Theoretical Physics in Santa Barbara on 'gravitational holography', which is precisely the theoretical study of quantum systems that lead to an emergent spacetime geometry. The workshop is exploring the idea that a sufficiently complex quantum system is a universe of its own.

There are still many unanswered questions regarding quantum black holes that hopefully will also be answered soon. Perhaps the most important question is the proper interpretation or resolution of the black hole singularity. What can we say about the observer that falls into the singularity? How do we describe physics in that region? These questions are important because the black hole singularity is somewhat similar to the singularity at the beginning of the Universe, just its time reversed, a big crunch rather than a big bang. In fact, the main motivation to study quantum aspects of black holes is to learn lessons that can then be applied to quantum aspects of cosmology. This is something that has already happened in the past: Hawking radiation has a cosmological analogue, the creation of primordial fluctuations during inflation. These are the fluctuations observed as small anisotropies of the cosmic microwave background. We hope that further understanding will allow us to explain other aspects of the Universe, such as the particular pattern of fields and interactions that we have in the standard model.

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Competing interests

The author declares no competing interest.